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COMMENT

The K matrix and the bar phase parameters

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Abstract. The elements of the **K** matrix are expressed in terms of the bar phase parameters. It is shown that an unnecessary constraint has been imposed in the recent phase-shift analysis of the neutron–proton scattering data since an approximation to the **K** matrix at low energies is not valid if one phase parameter takes the value $\frac{1}{2}\pi$.

The recent phase-shift analysis of the neutron–proton scattering data by Arndt *et al* (1977) gives rise to an unusual energy dependence of the mixing parameter ϵ in the $^3S_1$–$^3D_1$ state. In particular, ϵ is zero at two separate energies. However, the first zero, at an energy of about 18 MeV was imposed on the analysis as a result of a **K** matrix argument which appeared to suggest that it was a consequence of the first phase parameter taking the value of $\frac{1}{2}\pi$ radians. Counter examples are the Reid potentials (1968) for which Arndt's **K** matrix description obviously fails. We now present a mathematical discussion of this failure using explicit formulae for the **K** matrix elements in terms of the bar phase parameters.

The relations between the **S**, **T**, **K** and **M** matrices are given in the paper by Ross and Shaw (1960). Arndt *et al* considered the relation between the **T** and **K** matrices. We consider the relation between the **K** and **M** matrices since the expressions for the **M** matrix elements in terms of the bar phase parameters are relatively straightforward and have already been reported in the literature (Kermode 1967, McKerrell *et al* 1977). Hence, leaving off the bars for simplicity, we take as our starting point the expression

$$\mathbf{K} = A\begin{pmatrix} \cot\Delta_{22} & -\cot\Delta_{12} \\ -\cot\Delta_{12} & \cot\Delta_{11} \end{pmatrix}$$

where

$$A^{-1} = \cot\Delta_{11}\cot\Delta_{22} - \cot^2\Delta_{12},$$

$$\Delta_{11} = \delta_1 - \tan^{-1}(\tan^2\epsilon\cot\delta_2)$$

$$\Delta_{22} = \delta_2 - \tan^{-1}(\tan^2\epsilon\cot\delta_1)$$

and

$$\tan\Delta_{12} = -\cot\epsilon\sin\delta_1\sin\delta_2 + \tan\epsilon\cos\delta_1\cos\delta_2.$$

In relation to the notation used by Arndt *et al* for the particular case of $^3S_1$–$^3D_1$ scattering $\delta_1 = \delta_S$ and $\delta_2 = \delta_D$.

After a little algebra, we find that

$$K_{11} = (\tan\delta_1 - \tan^2\epsilon\cot\delta_2)\alpha/\gamma\beta_{12}$$

and

$$K_{12} = \tan\epsilon(1 - \tan^2\epsilon\cot\delta_1\cot\delta_2)/\gamma\cos\delta_1\cos\delta_2$$

where

$$\alpha = 1 + \tan^4\epsilon + \tan^2\epsilon(\tan\delta_1\cot\delta_2 + \tan\delta_2\cot\delta_1),$$

$$\beta_{12} = 1 + \tan^2\epsilon\tan\delta_1\cot\delta_2$$

and

$$\gamma = \alpha - \tan^2\epsilon\tan\delta_1\tan\delta_2\operatorname{cosec}^2\delta_1\operatorname{cosec}^2\delta_2.$$

The expression for K_{22} is obtained from that for K_{11} by interchanging the suffices 1 and 2.

For n–p, ${}^3S_1$–${}^3D_1$, scattering, we consider the situation at low energies where $\tan\delta_1$, ϵ and δ_2 are of order k, k^3 and k^5 respectively. Then α and β_{12} are $1 + O(k^2)$ and γ is of order unity. In fact we find

$$K_{11} = \tan\delta_1(1 + O(k^{10})) \qquad \text{and} \qquad \cos\delta_1 K_{12} = \tan\epsilon(1 + O(k^{10}))$$

in practical agreement with the results of Arndt *et al.* However, these authors use the approximate result to argue that when δ_1 passes through $\frac{1}{2}\pi$, ϵ changes sign provided that K_{12} is not also zero. The error in Arndt's argument lies in the application of an approximate formula in a region where the approximation is not valid, since $\tan\delta_1 \neq O(k)$. It is a simple matter to show that if $\delta_1 = \frac{1}{2}\pi$, then $-\tan\epsilon\sin\delta_2 K_{12} = 1$ which puts no constraint whatsoever on the mixing parameter ϵ, as expected since ϵ is independent of δ_1 and δ_2 and we also have the particular example of the Reid potential.

The constraint imposed by Arndt *et al* is not justified. However, it remains a possibility that nature prefers a zero value for ϵ at *or near* that of Arndt's so the results of the phase-shift analysis need not be rejected, although it would be useful to have an analysis without the constraint.

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